# PARAMETER IDENTIFICATION OF THE TWO-MASS MECHANICAL FLEXIBLE SYSTEM USING WELCH METHOD

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**Abstract**: This paper is focused on the parameter identification of the Two-Mass mechanical flexible system for motor drive applications. The whole methodology is based on the amplitude frequency characteristics given by Welch spectrum analysis method. The iterative algorithm is used to reconstruct the bi-quadratic flexible member of the transfer function. Then, the mechanical parameters are obtained with the transfer function parameters equivalence. Finally, the experimental results and theoretical estimates are compared and discussed.

**Keywords**: Two-mass flexible system, Welch method, Spectral analysis, Bi-quadratic filter

## 1 INTRODUCTION

The many cases of the mechanical drive systems used in the industry can be modeled as the two-mass system. As the examples; the toothed belt, axial rotary flexible connection, and long torque shaft connection can be given. Some knowledge background of the physical based modelling approach for the multi-mass systems can be found here [1]. This type of systems or plants shows resonant behavior on the certain frequencies. This means that there are resonant and anti-resonant peaks on the plant frequency response. Such a behavior causes many problems in closed-loop system design and hence, it is necessary to identify this behavior as best as possible. The system identification is often challenging task because of the choice of the right method and suitable input signal which is rich enough for good plant excitation. Widely used methods for identification of the electric drive systems are methods based on the frequency analysis or on the spectrum analysis. Method used in this paper is based on the spectral analysis using power spectral densities, often called periodograms. The main goal of the identifiation experiment presented in this paper is to gain the plant parameters in such a way, which allows success reconstruction of the resonant and anti-resonant peaks values and their positions on the plant frequency response.

## 2 TWO-MASS SYSTEM MODELLING

The Two-Mass mechanical flexible system can be taken as the system with single input and two outputs and can be described by the following set of the differential equations.

$$\frac{d\omega_m}{dt} = -\frac{b}{J_m}\omega_m - \frac{k}{J_m}\theta_m + \frac{b}{J_m}\omega_l + \frac{k}{J_m}\theta_l + \frac{T_i}{J_m} \tag{1}$$

$$\frac{d\theta_m}{dt} = \omega_m \tag{2}$$

$$\frac{d\omega_l}{dt} = \frac{b}{J_l}\omega_m + \frac{k}{J_l}\theta_m - \frac{b}{J_l}\omega_l - \frac{k}{J_l}\theta_l \tag{3}$$

$$\frac{d\theta_l}{dt} = \omega_l \tag{4}$$

Where $J_m$ and $J_l$ are the inertias of the shaft and load respectively. Parameter $b$ is the dumping coefficient and the parameter $k$ is the flexible coefficient. Variables $\omega_m$ and $\omega_l$ stand for shaft and load angular velocities respectively. The variables $\theta_m$ and $\theta_l$ are shaft and load angular positions respectively. Finally, the $T_i$ variable stands for the input torque generated by the electric part of the machine. Typical frequency response of the transfer function given as the ratio between the input torque and shaft angular velocity is at Figure 1.

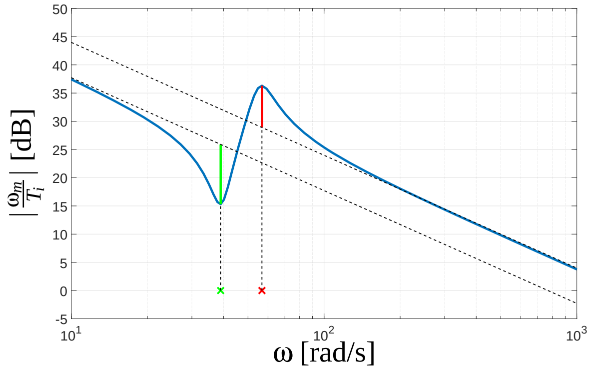


**Figure 1:** Flexible system frequency response (example)

The blue line represents magnitude of the mechanical part of the drive systems with flexible mechanical part. Red and green lines represent resonance and anti-resonance overshoots respectively. Red and green crosses then represent their positions in the frequency. Mentioned transfer function of the examined plant is as follows:

$$Gp(s) = \frac{\omega_m(s)}{T_i(s)} = \frac{1}{J_m + J_l} \frac{1}{s} \frac{J_l s^2 + bs + k}{J_m J_l s^2 + bs + k} \tag{5}$$

Where the $s$ stands for Laplace operator. The bi-quadratic member creates the flexible element of the system. The main goal of this paper is to show the procedure of the $J_m$, $J_l$, $k$, and $b$ parameters identification. The resonant angular frequency $\omega_r$ and anti-resonant angular frequency $\omega_a$ are described by the following relations [2], [3]:

$$\omega_r = \sqrt{k \frac{J_m + J_l}{J_m J_l}} \tag{6}$$

$$\omega_a = \sqrt{\frac{k}{J_l}} \tag{7}$$

The analytical form of the amplitude frequency characteristic can be easily derived by replacing $s$ with the $j\omega$ in (5) and evaluating the absolute value to gain the following relation:

$$|G_p(\omega)|_{dB} = 20 \log_{10} \sqrt{\frac{(k - J_l\omega^2)^2 + b^2\omega^2}{(k(J_m + J_l) - J_m J_l \omega^2)^2 + b^2(J_m + J_l)^2\omega^2}} \tag{8}$$

Now, the values of the resonant and anti-resonant peaks can be easily derived by substituting (6) and (7) into (8) respectively. Then, the following relations are obtained:

$$R_{dB} = 20\log_{10}\frac{1}{J_m+J_l}\sqrt{1+\frac{k\frac{J_l^3}{J_m}}{b^2(J_m+J_l)}} \tag{9}$$

$$A_{dB} = 20\log_{10}\sqrt{\frac{b^2}{b^2(J_m+J_l)^2+kJ_l^3}} \tag{10}$$

Relations (6), (7), (9), and (10) are used to examine the accuracy of the identified parameters in the final part of this paper.

## 3 DATA PROCESSING

As mentioned in the Introduction section, Welch method is used for the data processing. Whole algorithm can be expressed by the following equation:

$$|G_p(\omega)| = \frac{\mathscr{F}\frac{1}{N}\sum_{i=0}^{N} R_{uy_i}(\tau)w_i(\tau)}{\mathscr{F}\frac{1}{N}\sum_{i=0}^{N} R_{uu_i}(\tau)w_i(\tau)} \tag{11}$$

Where $\mathscr{F}$ stands for Fourier transform, $R_{uu_i}(\tau)$ and $R_{uy_i}(\tau)$ stand for i-th input auto correlation and input-output cross correlation functions respectively. The $w_i(\tau)$ is i-th window function. Fourier transform is applied on the average of the $N$ windowed correlated functions and FFT algorithm is used to evaluate the Fourier transform. The length of the window function $L_w$ (and windowed correlated functions as well) is set to 2048 samples. This is due to radix format for FFT algorithm. The length of the one period of PRBS is set to 1023 symbols. The length of correlated function is given by $2M-1$, where $M$ is length of the input data sequence for the correlation functions. Because of the attributes of PRBS, the whole period is correlated, which gives 2045 sample length. To reach the radix form, the correlated data sequence is filled with zeros.

Because of the result accuracy, the window function $w(\tau)$ is applied. Its prescription is given by:

$$w(\tau) = \begin{cases} \frac{1}{2}(1+cos\frac{\pi\tau}{P}) & |\tau| \le P \\ 0 & |\tau| > P \end{cases} \tag{12}$$

Which is called Hamming-Tukey window. Parameter $P$ stands for window span and it needs to be narrow enough to filter out noisy data from correlation functions but not too much narrow to cut off valid information from them. Window span is usually chosen from following interval [4]:

$$P \in \left\langle \frac{L_w}{6}, \frac{L_w}{5} \right\rangle \tag{13}$$

### 3.1 PARAMETERS EXTRACTION

For the parameters extraction it is necessary to reconstruct the transfer function of the plant (5). For this purpose an iterative algorithm was developed. This algorithm iteratively and alternately sets following polynomials:

$$\begin{aligned} \alpha(s) &= s^2 + 2\xi_\alpha\omega_\alpha + \omega_\alpha^2 \\ \beta(s) &= s^2 + 2\xi_\beta\omega_\beta + \omega_\beta^2 \end{aligned} \tag{14}$$

Altogether, these formulas form Bi-quadratic filter in the following form:

$$G_{BiQ}(s) = \frac{s^2 + 2\xi_\alpha \omega_\alpha + \omega_\alpha^2}{s^2 + 2\xi_\beta \omega_\beta + \omega_\beta^2} \tag{15}$$

Detailed description of the iterative algorithm can be found here [5]. Now, if the parameters of the (15) are matched with the parameters of the (5), the relations for the $J_m$, $J_l$, $k$, and $b$ parameters can be easily found.

## 4 EXPERIMENT DESCRIPTION

For the purpose of the experiment, the long torque shaft connected to the Permanent magnet synchronous motor (PMSM) was used. The inertia disc was placed at the other end of the shaft. The PMSM is drived by the field oriented control. The electric part of the machine is controlled via current PI controllers. The sampling frequency for the measurement was set to 1 kHz. The q-axis current and the shaft velocity were measured and used for the spectral analysis. The measured velocity signal was derived to omit the integral character of the mechanical plant. Number of PRBS periods was set to 50. The window span is set as follows: $P = \frac{2048}{5.5}$. The complete algorithm was implemented at dSPACE DS1103 platform. In Figure 2, the measurement data can be seen together with the gained amplitude frequency characteristic of the mechanical plant.

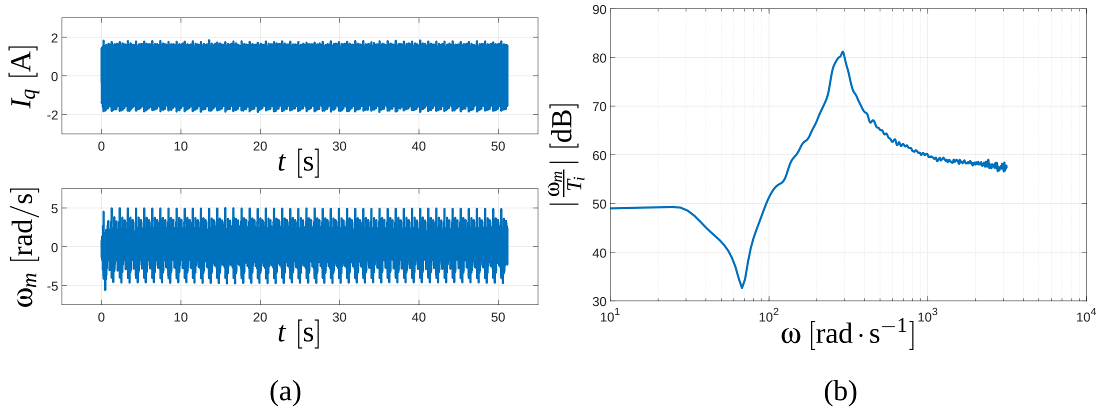


**Figure 2:** (a) Measured data - q-axis current (upper), motor shaft velocity (lower), (b) amplitude frequency characteristic given by Welch spectrum analysis

There are measured and reconstructed $\omega_r$, $\omega_a$, $R_{db}$, and $A_{dB}$ values to compare together with their relative percentage errors highlighted in the following table:

| Parameter | measured value | reconstructed value | error |
|---|---|---|---|
| $\omega_r$ | 294.7 rad·s$^{-1}$ | 306.7 rad·s$^{-1}$ | 4.1 % |
| $\omega_a$ | 67.5 rad·s$^{-1}$ | 67.8 rad·s$^{-1}$ | 0.5 % |
| $R_{dB}$ | 80.4 dB | 78.9 dB | 1.9 % |
| $A_{dB}$ | 32.6 dB | 32.7 dB | 0.3 % |

**Table 1:** Parameters comparison

As one can see, the most problematic parameter to estimate is the position of the resonance peak in the amplitude frequency characteristic. This is due to the dynamics of the electrical part of the plant, where its dynamic is situated approximately at 570 rad·s$^{-1}$. The characteristic at Figure 2 (b) is modified by subtraction following function which represent electrical part of the PMSM. This is because original measurement was influenced by the electrical part of the PMSM.

$$|G_e(\omega)| = 20\log_{10}\frac{1}{\sqrt{1+(T_e\omega)^2}} \tag{16}$$

Where $T_e$ is the time constant of the PMSM and its plain to see thus $\frac{1}{T_e} = 570\ \mathrm{s}^{-1}$. The estimated mechanical plant parameters can be found in Table 2.

| Parameter | value |
|---|---|
| $J_m$ | $1.7 \cdot 10^{-4}\ \mathrm{k \cdot gm^2}$ |
| $J_l$ | $3.3 \cdot 10^{-3}\ \mathrm{kg \cdot m^2}$ |
| $k$ | $15\ \mathrm{m^2 \cdot kg \cdot s^{-2}}$ |
| $b$ | $0.03\ \mathrm{m^2 \cdot s^{-1}}$ |

**Table 2:** Estimated mechanical plant parameters

## 5 CONCLUSION

The main goal of this paper was to demonstrate the possible use of Welch spectrum analysis for the motor drive plants with the two-mass mechanical resilient load. A procedure for mechanical plant parameters extraction was suggested. A comparison between measured and estimated resonance and anti-resonance behavior was done. The highest estimation error is caused by the electrical part of the drive machine, because the dynamics of the electrical part is situated close to the resonance peak in the amplitude frequency characteristic. However, the relative percentage errors of the parameters estimates are satisfying low. This allows to use these estimates for the potential controlled closed-loop system design.


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